Comparison of algorithms for incoming atmospheric long-wave radiation

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While numerous algorithms exist for predicting incident atmospheric long-wave radiation under clear ($L_{clr}$) and cloudy skies, few comparisons have been published to assess the accuracy of the different algorithms. Virtually no comparisons have been made for both clear and cloudy skies across multiple sites. This study evaluates the accuracy of 13 algorithms for predicting incident long-wave radiation under clear skies, ten cloud correction algorithms, and four algorithms for all-sky conditions using data from 21 sites across North America and China. Data from five research sites were combined with publicly available data from nine sites in the AmeriFlux network for initial evaluation and optimization of cloud cover estimates; seven additional AmeriFlux sites were used as an independent test of the algorithms. Clear-sky algorithms that excelled in predicting $L_{clr}$ were the Dilley, Prata, and Ångström algorithms. Root mean square deviation (RMSD) between predicted and measured 30-minute or hourly $L_{clr}$ averaged approximately 23 W m$^{-2}$ for these three algorithms across all sites, while RMSD of daily estimates was as low as 14 W m$^{-2}$. Cloud-correction algorithms of Kimball, Unsworth, and Crawford described the data best when combined with the Dilley clear-sky algorithm. Average RMSD across all sites for these three cloud corrections was approximately 24 to 25 W m$^{-2}$ for 30-minute or hourly estimates and approximately 15 to 16 W m$^{-2}$ for daily estimates. The Kimball and Unsworth cloud corrections require an estimate of cloud cover, while the Crawford algorithm corrects for cloud cover directly from measured solar radiation. Optimum limits in the clearness index, defined as the ratio of observed solar radiation to theoretical terrestrial solar radiation, for complete cloud cover and clear skies were suggested for the Kimball and Unsworth algorithms. Application of the optimized algorithms to seven independent sites yielded similar results. On the basis of the results, the recommended algorithms can be applied with reasonable accuracy for a wide range of climates, elevations, and latitudes.


1. Introduction

Downwelling long-wave radiation is vitally important for numerous applications requiring surface radiation and energy balance, including predicting evapotranspiration, snowmelt, surface temperature, and frost occurrence. Meteorological stations rarely include long-wave radiation sensors, so measurements of incoming long-wave radiation are generally not available for a given location and period of interest. While surface radiative fluxes can be calculated with reasonable accuracy using complex radiative transfer models [Edwards and Slingo, 1996; Pope et al., 2000], these require detailed measurements of the air column above a site, including cloud properties and vertical profiles of temperature, water vapor, trace gases, and aerosols. Because these measurements are rarely available, downwelling long-wave radiation is commonly estimated from algorithms that use more readily available meteorological observations, such as air temperature, humidity, and solar radiation. Although these simpler algorithms may have larger errors relative to the more complex methods, these methods are needed and are useful for a variety of disciplinary applications, despite the small errors.

Several parameterizations have been developed that produce estimates for downwelling long-wave radiation ($L_d$) using synoptic observations [e.g., Idso and Jackson, 1969; Maykut and Church, 1973; Jacobs, 1978; Idso, 1981; Aubinet, 1994; Dilley and O’Brien, 1998; Duarte et al., 2006; Lhomme et al., 2007]. However, the few studies that offer a comparison of the different algorithms against measured $L_d$ do not include more than two or three sites [Alados-Alboledas et al., 1995; Orsini et al., 2000; Niemelä et al., 2001; Iizomon et al., 2003; Pérez-Garcia, 2004]. Hatfield et al. [1983] and Finch and Best [2004] did evaluate algorithms for multiple locations. Hatfield et al. [1983] compared several algorithms for clear skies at 15
locations across the United States with measured \( L_d \); Finch and Best [2004] compared five algorithms for \( L_d \) against values predicted by the radiation transfer scheme within a climate model to determine their suitability for global applications. However, Finch and Best [2004] did not use measured \( L_d \) and neither of these comparisons considered any of the available cloud cover corrections.

[4] Broad-scale comparisons of \( L_d \) algorithms are not available, in part, because of the relative scarcity of data sets that include measurements of \( L_d \). However, the availability of AmeriFlux data make \( L_d \) measurements available for numerous sites, representing a broad range of climates. The purpose of this paper was to: compare a variety of algorithms for estimating clear and cloudy sky incoming long-wave radiation across numerous sites representing a variety of climates; determine which algorithms work best; and evaluate the error for different algorithms.

2. Parameterization Schemes

[5] Downwelling long-wave radiation is the thermal radiation emitted by the atmosphere downward to the ground surface. It is often computed from a general equation following the form of Stefan-Boltzmann equation.

\[
L_d = \varepsilon_{\text{eff}} \sigma T_{\text{eff}}^4
\]

where \( \varepsilon_{\text{eff}} \) and \( T_{\text{eff}} \) (K) are the effective emissivity and temperature of the atmosphere above the site, and \( \sigma \) is the Stefan-Boltzmann constant. The temperature and emissivity are integrated quantities over an atmospheric column above the site and are typically estimated from ground-based meteorological observations only.

2.1. Clear-Sky Parameterizations


\[
L_{clr} = \varepsilon_{\text{clr}} \sigma T_o^4
\]

where \( L_{clr} \) is the incoming long-wave radiation for clear skies, \( \varepsilon_{\text{clr}} \) is the clear-sky emissivity, and \( T_o \) is near-surface air temperature (K). Several algorithms have been developed to estimate downward surface long-wave radiation flux using synoptic observations only. We assessed 13 algorithms given in Table 1 for estimating clear-sky downwelling long-wave flux (\( L_{clr} \)). In this article, the unit of the radiative flux is W m\(^{-2}\), the unit of vapor pressure (\( e_o \)) is kPa, and the unit of \( T_o \) is K.

2.2. Elevation Corrections

[7] Elevation can affect \( L_d \) because the air column above the site decreases with elevation. Deacon [1970] developed an elevation correction based on the clear-sky equation of Swinbank [1963]. It is expressed as

\[
L_{clr-elev} = L_{clr} - 0.035 \left( \frac{z}{1000} \right) \sigma T_o^4
\]

where \( z \) is elevation of the site in meters a.m.s.l., however this expression was developed using data to no more than 1700 m.


\[
\varepsilon_{\text{clr}} = 1.723 \left( \frac{e'}{T'} \right)^{1/7} \times \left( \frac{P_o}{P_{sl}} \right).
\]

Here, \( e' \) and \( T' \) are vapor pressure and temperature adjusted to their sea level equivalents assuming a standard lapse rate and constant humidity, \( P_o \) is air pressure at the site and \( P_{sl} \) is sea level air pressure (101.3 kPa). This expression was

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Table 1. Algorithms for Estimating Clear-Sky Emissivity Following the Form of the Stefan-Boltzmann Equation or for Estimating Downwelling Long-Wave Radiation Directly*  

<table>
<thead>
<tr>
<th>Source</th>
<th>Clear-Sky Algorithm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ångström [1918]*</td>
<td>( \varepsilon_{\text{clr}} = 0.83 - 0.18 \times 10^{-0.007e_o} )</td>
</tr>
<tr>
<td>Brunt [1932]*</td>
<td>( \varepsilon_{\text{clr}} = 0.52 + 0.205/y )</td>
</tr>
<tr>
<td>Brutsaert [1975]</td>
<td>( \varepsilon_{\text{clr}} = 1.723 \left( \frac{e}{T} \right)^{1/7} )</td>
</tr>
<tr>
<td>Garratt [1992]</td>
<td>( \varepsilon_{\text{clr}} = 0.79 - 0.17 \exp(-0.96e_o) )</td>
</tr>
<tr>
<td>Idso and Jackson [1969]; referred to as Idso-1</td>
<td>( \varepsilon_{\text{clr}} = 1 - 0.261 \exp(-0.0007(T_o - 273.16)) )</td>
</tr>
<tr>
<td>Idso [1981]; referred to as Idso-2</td>
<td>( \varepsilon_{\text{clr}} = 0.7 + 5.95 \times 10^{-4}e_o \exp\left(\frac{1500}{T_o}\right) )</td>
</tr>
<tr>
<td>Iziomon et al. [2003]*</td>
<td>( \varepsilon_{\text{clr}} = 1 - X \exp\left(\frac{500}{T_o}\right) )</td>
</tr>
<tr>
<td>Keding [1989]</td>
<td>( \varepsilon_{\text{clr}} = 0.92 - 0.7 \times 10^{-1.2e_o} )</td>
</tr>
<tr>
<td>Niemelä et al. [2001]</td>
<td>( \varepsilon_{\text{clr}} = \begin{cases} 0.72 + 0.09(e_o - 0.2) &amp; \text{for } e_o \geq 0.2 \ 0.72 - 0.76(e_o - 0.2) &amp; \text{for } e_o &lt; 0.2 \end{cases} )</td>
</tr>
<tr>
<td>Prata [1996]*</td>
<td>( \varepsilon_{\text{clr}} = 1 - (1 + w) \exp(-1.23 + 3w)^{1/2} )</td>
</tr>
<tr>
<td>Satterlund [1979]</td>
<td>( \varepsilon_{\text{clr}} = 1.08\left[1 - \exp(-10e_o)^{1/2016}\right] )</td>
</tr>
<tr>
<td>Swinbank [1963]</td>
<td>( L_{clr} = 5.31 \times 10^{11}T_o^{6} + 96.96\sqrt{w/25} )</td>
</tr>
<tr>
<td>Dilley and O’Brien [1998]*</td>
<td>( L_{clr} = 59.38 + 113.7 \frac{T_o^{6}}{273.16} + 96.96\sqrt{w/25} )</td>
</tr>
</tbody>
</table>

*Coefficients are based on vapor pressure (\( e_o \)) in kilopascals, temperature (\( T_o \)) in kelvins, and precipitable water (\( w \)) in centimeters.

As cited by Niemelä et al. [2001].

\( w_o = 4650e_o/T_o \); Prata, 1996.

Values for \( X \) and \( Y \) in the algorithm of Iziomon et al. [2003] were interpolated between a lowland site at 212-m elevation (\( X = 0.35 \) and \( Y = 100 \) kPa\(^{-1} \)) and a mountain site at 1489-m elevation (\( X = 0.43 \) and \( Y = 115 \) K kPa\(^{-1} \)).
developed using periodic clear-sky data from 26 sites ranging in elevation from 2929 to 3750 m across the southern Sierra Nevada Mountains of California.

[9] Iziomon et al. [2003] reported different parameter values for their algorithm based on sites at different elevations. Herein, parameters for the Iziomon algorithm were extrapolated from those reported for a lowland site and a mountain site (Table 1).

### 2.3. Estimates of Cloud Cover

[10] Clear-sky long-wave radiation computed from algorithms in Table 1 must be adjusted for cloud cover. Most approaches adjust \( \varepsilon_{clr} \) for the fraction of cloud cover, \( c \), to compute an effective atmospheric emissivity, \( \varepsilon_a \). Cloudiness can be estimated from a clearness index (\( k \)), defined herein as the ratio of solar radiation flux density (\( S_o \)) to total hemispherical solar radiation flux density incident on a horizontal surface at the outer edge of the earth’s atmosphere (\( S_o \)). Campbell [1985] suggests that \( c \) can be linearly interpolated between \( c = 1.0 \) at a clearness index of 0.4 for complete cloud cover (\( k_{clcr} \)) to \( c = 0.0 \) at a clearness index of 0.7 (\( k_{clcl} \)). Others have used values of 0.35 and 0.6 for \( k_{clcr} \) and \( k_{clcl} \) respectively [Flerchinger, 2000; Xiao et al., 2006]; the optimum values are unknown and may perhaps depend on the location.


\[
S_o = S_{clr}(1 - b_1c^{b_2})
\]

where \( S_{clr} \) is incoming short-wave radiation for clear-sky conditions. They obtained values of 0.75 and 3.4 for \( b_1 \) and \( b_2 \) using data recorded in Hamburg, Germany over a 10-year period. The above equation can be rearranged to compute cloud cover by

\[
c = \left( \frac{1 - k_{clr}}{b_1} \right)^{1/b_2} \]

where \( k_{clr} \) is the clearness index for clear-sky conditions.

[12] The period over which solar radiation is accumulated for calculating \( k \) poses a problem for estimates at nighttime and low sun angles. Instantaneous readings are not usable at night, and small errors in solar radiation measurements at low sun angles can significantly bias estimates of \( k \). Additionally, daytime 30-minute or hourly solar readings may not necessarily be representative as clouds covering the solar disk may not be indicative of the entire hemispherical cloud cover. While some have limited their analyses to daytime observations [Crawford and Duchon, 1999], others have used average clearness estimates over a 2.5-hour window during the previous midafternoon for nighttime values [Lhomme et al., 2007], or solar radiation averaged over the entire day of observation [Flerchinger, 2000].

### 2.4. Cloudy-Sky Corrections

[13] Ten cloud-correction algorithms presented in Table 2 were assessed. Most algorithms adjust \( \varepsilon_{clr} \) for cloud cover to compute \( \varepsilon_a \). Because the clear-sky equations of Swinbank [1963] and Dilley and O’Brien [1998] are not based on a clear-sky emissivity (Table 1), it was necessary to back-calculate \( \varepsilon_{clr} \) from their estimates of \( L_{clr} \) before applying the equations in Table 2. To avoid the complication of estimating cloud cover, Crawford and Duchon [1999] and Lhomme et al. [2007] based their cloud corrections on the ratio of measured solar radiation (\( S_o \)) to the clear-sky irradiance (\( S_{o,c} \)): \( s = S_o/S_{o,c} \), referred to herein as a solar index. The approach used by Crawford and Duchon [1999] for estimating \( S_{o,c} \) for a given site and time of year is given in Appendix A.

[14] Kimball et al. [1982] presented a cloud cover correction for multiple cloud layers within the atmosphere. Herein, Kimball’s approach was simplified to a single cloud layer with a fractional cover \( c \), emissivity of 1.0, and temperature \( T_e \). For this exercise, \( T_e \) was assumed to average 11 K cooler than air temperature at the surface with a seasonal variation of \( \pm 2 \) K, higher in summer than in winter [Unsworth and Monteith, 1975]. While Kimball et al. [1982] indicates that thin cloud types, such as cirrus, have emissivities averaging 0.5, their solar transmissivity will

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**Table 2.** Algorithms for Correcting Clear-Sky Emissivity or Clear-Sky Long-Wave Radiation for Cloudy Skies

<table>
<thead>
<tr>
<th>Source</th>
<th>Cloud-Correction Algorithm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brutsaert [1982]</td>
<td>( \varepsilon_a = (1 + 0.22c)\varepsilon_{clr} )</td>
</tr>
<tr>
<td>Iziomon et al. [2003]*</td>
<td>( \varepsilon_a = (1 + 0.27c)\varepsilon_{clr} )</td>
</tr>
<tr>
<td>Jacobs [1978]</td>
<td>( \varepsilon_a = (1 + 0.26c)\varepsilon_{clr} )</td>
</tr>
<tr>
<td>Keding [1989]</td>
<td>( \varepsilon_a = (1 + 0.153c^{0.148})\varepsilon_{clr} )</td>
</tr>
<tr>
<td>Maykut and Church [1973]</td>
<td>( \varepsilon_a = (1 + 0.22c^{0.75})\varepsilon_{clr} )</td>
</tr>
<tr>
<td>Sugita and Brutsaert [1993]</td>
<td>( \varepsilon_a = (1 + 0.0496c^{0.45})\varepsilon_{clr} )</td>
</tr>
<tr>
<td>Unsworth and Monteith [1975]</td>
<td>( \varepsilon_a = (1 - 0.84c)\varepsilon_{clcr} + 0.84c )</td>
</tr>
<tr>
<td>Kimball et al. [1982]</td>
<td>( L_d = L_{clr} + \tau_s\varepsilon_d\sigma T_e^4 )</td>
</tr>
<tr>
<td></td>
<td>( \tau_s = 1 - \varepsilon_s(1.4 - 0.4\varepsilon_s) )</td>
</tr>
<tr>
<td></td>
<td>( \varepsilon_s = 0.24 + 2.98 \times 10^{-7}e_o^2 \exp(\frac{3000}{T_o}) )</td>
</tr>
<tr>
<td></td>
<td>( f_s = 0.6732 + 0.6240 \times 10^{-5}T_e^2 - 0.01940 \times 10^{-5}T_e^2 )</td>
</tr>
</tbody>
</table>

*Values for \( Z \) in the algorithm of Iziomon et al. [2003] were extrapolated from values \( Z = 0.35 \) for a site at 212-m elevation and 0.50 for a site at 1489-m elevation.
also be higher, resulting in an underestimate of cloud cover based on $k$ or $s$. These errors will tend to compensate for each other, resulting in relatively small errors in assuming 1.0 for cloud emissivity.

Four additional schemes for directly estimating downward long-wave radiation of variably cloudy skies introduced by Aubinet [1994] were also evaluated. These are presented in Table 3. Each of the algorithms of Aubinet [1994] uses a different combination of temperature, vapor pressure and clearness index depending on data availability.

### 3. Methods

Data collected from five separate sites were combined with nine AmeriFlux sites to develop a data set for initial evaluation and optimization of the long-wave radiation algorithms. Seven additional AmeriFlux sites were used for an independent test of the algorithms. In most cases, two years of data were used for analysis. Basic climatic characteristics of each site are summarized in Table 4.

The Idaho site is Reynolds Mountain located on the Reynolds Creek Experimental Watershed operated by the USDA Agricultural Research Service. The site is semiarid rangeland characterized by a Mediterranean climate. Data were used from three sites in China. The Yucheng Comprehensive Experiment Station of the Chinese Academy of Sciences lies on the North China Plain and is subject to a monsoonal climate. Measurements were collected at Yucheng above an irrigated wheat/maize field. Ganzi and Nagqu are two high-elevation semiarid grassland sites on the Tibetan Plateau. Data from Iowa were collected on the Brooks field near Ames, IA, a humid midcontinental site. Like most of the AmeriFlux sites, temperature and humidity at the Idaho, Iowa and Yucheng sites were collected using an HMP45 humidity probe (Vaisala, Helsinki, Finland), and incoming solar and long-wave radiation at the Iowa and China sites were measured using an unventilated, unheated, four-component net radiometer (CNR-1, Kipp and Zonen, Delft, Netherlands), calibrated every couple years. A newly calibrated CG3 pyrgeometer, which is the same sensor used for long-wave radiation in the CNR-1, was installed in 2003 at the Idaho site. Data from Idaho and Iowa are 30-minute averages, and data from Yucheng, China are hourly averages. Michel et al. [2008] reported accuracies of 16 W m$^{-2}$ for hourly averages and 12 W m$^{-2}$ for daily averages for the downward long-wave component of unheated, unventilated CNR-1 radiometers. Radiation data from Ganzi and Nagqu were collected using a pyrheliometer (MS-52, EKO, Japan) and an Eppley Precision Infrared Radiometer calibrated prior to the experiment and comparatively checked afterward. Temperature and humidity were measured using an aspirated psychrometer.

Data from the AmeriFlux sites were downloaded from the Web interface located at http://public.ornl.gov/ameriflux/. Sites were screened using the graphing interface for those sites that collect incoming long-wave radiation and have two years of relatively high-quality temperature, humidity, solar radiation and long-wave radiation data, while trying to cover a broad range of climates and geographic regions. The sites selected for initial evaluation

### Table 3. Algorithms for Directly Computing Long-Wave Radiation for All-Sky Conditions Based on Vapor Pressure ($e_v$, in Kilopascals), Temperature ($T_a$ in Kelvins), and Clearness Index ($k$)

<table>
<thead>
<tr>
<th>Name</th>
<th>$L_d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aubinet-1</td>
<td>$L_d = 69 - 1.97T_a - 19.90$</td>
</tr>
<tr>
<td>Aubinet-2</td>
<td>$L_d = 0.341 + 0.133 \ln[1 - k] - 0.341T_a$</td>
</tr>
<tr>
<td>Aubinet-3</td>
<td>$L_d = 0.341 + 0.133 \ln[1 - k] - 0.341T_a$</td>
</tr>
<tr>
<td>Aubinet-4</td>
<td>$L_d = 0.682 + 0.0352 \ln[1 - k] + 0.133 \ln[1 - k] - 0.341T_a$</td>
</tr>
</tbody>
</table>

### Table 4. Characteristics of Sites Used for Testing Algorithms

<table>
<thead>
<tr>
<th>Location</th>
<th>Latitude/Longitude</th>
<th>Elevation (m)</th>
<th>Average Air Temperature (°C)</th>
<th>Average Vapor Deficit (kPa)</th>
<th>Average Clearness Index</th>
<th>Average $L_d$ (W m$^{-2}$)</th>
<th>Period</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arizona</td>
<td>31°49’N 110°52’W</td>
<td>1120</td>
<td>19.7</td>
<td>1.83</td>
<td>0.66</td>
<td>325</td>
<td>Jan 2004 to Dec 2005</td>
</tr>
<tr>
<td>British Columbia</td>
<td>49°52’N 125°20’W</td>
<td>300</td>
<td>8.0</td>
<td>0.26</td>
<td>0.40</td>
<td>312</td>
<td>Jan 2000 to Dec 2001</td>
</tr>
<tr>
<td>Colorado</td>
<td>40°02’N 105°33’W</td>
<td>3050</td>
<td>3.0</td>
<td>0.50</td>
<td>0.60</td>
<td>259</td>
<td>Jan 2000 to Dec 2001</td>
</tr>
<tr>
<td>Ganzi, China</td>
<td>31°37’N 100°00’E</td>
<td>3394</td>
<td>6.7</td>
<td>0.58</td>
<td>0.63</td>
<td>281</td>
<td>Aug 1982 to Jul 1983</td>
</tr>
<tr>
<td>Idaho</td>
<td>43°04’N 116°45’W</td>
<td>2097</td>
<td>6.1</td>
<td>0.62</td>
<td>0.59</td>
<td>274</td>
<td>Jan 2005 to Dec 2006</td>
</tr>
<tr>
<td>Indiana</td>
<td>39°19’N 86°25’W</td>
<td>285</td>
<td>12.1</td>
<td>0.44</td>
<td>0.49</td>
<td>329</td>
<td>Jan 2003 to Dec 2004</td>
</tr>
<tr>
<td>Iowa</td>
<td>41°58’N 96°42’W</td>
<td>313</td>
<td>11.7</td>
<td>0.55</td>
<td>0.54</td>
<td>322</td>
<td>Jun 2005 to Dec 2006</td>
</tr>
<tr>
<td>Mississippi</td>
<td>34°15’N 89°58’W</td>
<td>70</td>
<td>6.2</td>
<td>0.49</td>
<td>0.52</td>
<td>337</td>
<td>Jan 2004 to Dec 2005</td>
</tr>
<tr>
<td>Nagqu, China</td>
<td>31°29’N 92°03’E</td>
<td>4505</td>
<td>6.1</td>
<td>0.44</td>
<td>0.58</td>
<td>258</td>
<td>Aug 1982 to Jul 1983</td>
</tr>
<tr>
<td>North Carolina</td>
<td>35°59’N 79°6’W</td>
<td>163</td>
<td>15.0</td>
<td>0.57</td>
<td>0.52</td>
<td>327</td>
<td>Jan 2004 to Dec 2005</td>
</tr>
<tr>
<td>South Dakota</td>
<td>44°9’N 130°39’W</td>
<td>1750</td>
<td>6.1</td>
<td>0.59</td>
<td>0.58</td>
<td>276</td>
<td>Jan 2004 to Dec 2005</td>
</tr>
<tr>
<td>Washington</td>
<td>45°49’N 121°57’W</td>
<td>371</td>
<td>10.0</td>
<td>0.41</td>
<td>0.47</td>
<td>324</td>
<td>Jan 2003 to Dec 2004</td>
</tr>
<tr>
<td>Wisconsin</td>
<td>45°48’N 90°05’W</td>
<td>520</td>
<td>6.7</td>
<td>0.15</td>
<td>0.49</td>
<td>306</td>
<td>Jan 2004 to Dec 2005</td>
</tr>
<tr>
<td>Yucheng, China</td>
<td>36°50’N 116°34’E</td>
<td>28</td>
<td>16.0</td>
<td>0.63</td>
<td>0.43</td>
<td>338</td>
<td>Jun 2003 to Nov 2004</td>
</tr>
</tbody>
</table>

**Sites Used for Initial Evaluation and Optimization**

**Sites Used for Independent Testing**

Averages represent the data used for analysis and may not reflect annual averages due to data gaps and incomplete years.
included: Santa Rita, Arizona; Campbell River, British Columbia; Niwot Ridge, Colorado; Morgan Monroe State Forest, Indiana; Goodwin Creek, Mississippi; Duke Forest (pine site), North Carolina; Black Hills, South Dakota; Wind River Crane Site, Washington; and Willow Creek, Wisconsin. Seven additional sites used as an independent test of the optimized clearness indices included: Ivotuk, Alaska; Flagstaff, Arizona; Bondville, Illinois; Missouri Ozark; Fort Peck, Montana; Bismark, South Dakota; and Walker Branch, Tennessee. Measurements at all sites were at least two meters above the vegetation. Data from the sites were 30-minute averages, except the Indiana site had hourly data.

Feedback obtained from investigators at 12 of the 16 AmeriFlux sites indicated that most of the radiometers were calibrated annually, except the Arizona and British Columbia sites were calibrated approximately every two years. Topographic influence on incoming radiation was judged to be minimal or nonexistent based on investigator response and independent evaluation of the surrounding topography. Topography was most significant at the Washington site, with a large isolated hill located to the east extending approximately 13° above the horizon and a ridge to the south at approximately 9° above the horizon. The British Columbia site was located on a 6 to 9° east-facing slope. Niwot Ridge to the northwest of the Colorado site and the Santa Rita Mountains to the south of the Santa Rita, Arizona site extended approximately 8° above the horizon.

For our initial analyses, a clearness index for each observation was computed based on a 24-hour window of observed incoming solar radiation centered on the observation. The influence of the length of the time over which solar observations were used to compute the clearness index was subsequently assessed. Clear-sky algorithms were initially evaluated without elevation correction (with the exception of interpolating for the Iziomon parameters) using observations when the clearness index was greater the 0.70 based on the range in clearness index suggested by Campbell [1985]. Although some cloud corrections were developed specifically for a certain clear-sky algorithm, all ten cloud corrections were paired with all 13 clear-sky algorithms to ascertain the best combination of algorithms for all-sky conditions. In some cases, this may have violated assumptions regarding maximum clear-sky emissivity and resulted in εa values greater than 1.0, suggesting that the coupling was incompatible. However, by matching all cloud corrections with all clear-sky algorithms, the best combinations could be identified, and any mismatching would presumably result in poorer model results.

Model results were evaluated for mean bias difference (MBD), root mean square deviation (RMSD) and standard deviation (SD) of the 30-minute or hourly predictions, as well as RMSD of the predicted daily average fluxes (RMSDdaily). Ideally MBD and RMSD should approach zero and SD should mimic that of the observed values. RMSD can be separated into systematic (RMSDq) and unsystematic (RMSDc) errors. The systematic mean square deviation is evaluated as [Willmott, 1982]

\[ MSE_q = N^{-1} \sum_{i=1}^{N} \left( \hat{P}_i - O_i \right)^2 \]  

where \( \hat{P}_i \) is obtained from the least squares linear regression between the predicted variable, \( P_t \), and the observed variable, \( O_i \), i.e., \( \hat{P}_i = a + bO_i \). The unsystematic mean square deviation can be computed by difference: \( MSE_c = MSE - MSE_q \). High RMSDc in relation to RMSD implies that the form of the model or algorithm could capture much of the observed variation after a simple linear correction to remove bias (perhaps by changing a coefficient). RMSDc is a measure of how accurate the model could be without a more complex nonlinear adjustment to the form of the model.

The RMSD and SD of different algorithms were compared using an F test. Because the residuals for 30-minute and hourly estimates are strongly correlated, the number of independent samples is considerably less than the number of observations. Effective sample sizes were computed from the expression given by Lee and Lund [2004] adjusting for the lag-one autocorrelation of the residual time series.

\[ n_e = n \left( 1 - r(1) - 0.68n^{-1/2} \right) / \left( 1 + r(1) + 0.68n^{-1/2} \right) \]  

Here, \( n_e \) is the effective independent sample size for a given algorithm applied to a given site, \( n \) is the total number of observations for the site, and \( r(1) \) is the lag-one autocorrelation of the times series of the residuals between measurements and the estimates. On the basis of this, each site typically had one to two independent samples for every 24 hours of observation.

4. Results

4.1. Clear-Sky Algorithms

RMSD and MBD for each of the clear-sky algorithms without considering elevation corrections are given in Tables 5 and 6 for the initial 14 sites. The Ångström, Dilley and Prata algorithms stand out as being the best overall algorithms. The only site that the RMSD of the Prata algorithm was significantly higher (p < 0.05) than the best algorithm for that site was North Carolina; however, it was not significantly different at the p < 0.01 level. RMSD for the Ångström algorithm was significantly higher at p < 0.05 for the Arizona and North Carolina sites, but was not significant at p < 0.01. RMSD of the Dilley algorithm was significantly higher at p < 0.01 for the Arizona site. Except for the Yucheng, Mississippi and North Carolina sites, for which all algorithms overestimated \( L_{clr} \), the Dilley algorithm tended to underestimate \( L_{clr} \). Table 6), although its RMSD was among the best. Because of this bias, the systematic error (RMSDq) of the Dilley algorithm was significantly higher than the Ångström and Prata algorithms. Conversely, its unsystematic error (RMSDc) was the lowest, suggesting that it could perform best if the linear systematic error could be corrected. However, SD of the Dilley-predicted values is significantly different from the observed values, whereas the Ångström and Prata algorithms mimic the variability of the observed values quite well. Unsystematic error of the Garratt algorithm is similar to that of the Dilley algorithm, however its large negative MBD (Table 6) contributed to high RMSD and RMSDc.
Clear-sky long-wave radiation was most accurately predicted for the Arizona site, which also had the most number of clear days. RMSD was not significantly higher for any of the algorithms for the Yucheng site, due in part to the small number of clear days. RMSD was highest for the North Carolina and Mississippi sites; with the exception of the Brunt and Garratt algorithms, which consistently underestimated $L_{clr}$, all algorithms overestimated $L_{clr}$ for these sites. Algorithms tended to also overestimate $L_{clr}$ for the Yucheng site.

Idso-1 and Swinbank are the only algorithms that use only temperature to estimate $L_{clr}$. If humidity measurements are not available, there is no significant difference in RMSD between these two algorithms, however the systematic and bias errors of the Idso-1 algorithm are significantly higher than the Swinbank algorithm. An $F$ test of the RMSD$_{d}$ and a paired $t$ test of the MBD indicates that the Swinbank algorithm has a smaller MBD (5.3 W m$^{-2}$) and RMSD$_{d}$ (14.0 W m$^{-2}$) than the Idso-1 algorithm (MBD = +13.2 W m$^{-2}$; RMSD$_{d}$ = 17.7 W m$^{-2}$).

4.2. Elevation Corrections

The data indicate little if any influence of elevation on predicted $L_{clr}$. In fact, correlations between MBD in Table 6 and elevation imply a negative slope, whereas without elevation correction, overprediction of $L_{clr}$ is expected to increase with elevation. Ironically the highest elevation site, Ganzi, was among the sites that had the worst underprediction. The elevation correction developed by Marks and Dozier [1979] for the Brutsaert algorithm reduced estimated $L_{clr}$ by an average of 1.1 W m$^{-2}$ for the Washington site to 40.1 W m$^{-2}$ for the Nagu site. However, the Brutsaert algorithm already underpredicted $L_{clr}$ for nearly all sites (Table 6), so the elevation correction only resulted in further underprediction. The Deacon [1970] elevation correction suggests a slope of 12 W m$^{-2}$ per 1000 meters of elevation, depending on temperature. If the three sites that were consistently overpredicted (Yucheng, Mississippi and North Carolina) are ignored, the slopes of the MBD of Angström, Dilley and Prata algorithms with
elevation are essentially zero. Therefore an elevation correction was not considered further.

4.3. Estimates of Cloud Cover

[27] Algorithms presented by Campbell [1985] and Kasten and Czeplak [1980] for estimating cloud cover were compared. Each of the clear-sky algorithms in Table 1 was combined with each cloud-correction algorithm in Table 2 using both the cloud-estimation approaches of Campbell [1985] and Kasten and Czeplak [1980]. The RMSD for each combination is given in Table 7 using the approach of Campbell [1985]. The analyses in Table 7 assumed cloud cover ranged from 1.0 to 0.0 for a clearness index between $k_{clr} = 0.4$ and $k_{clr} = 0.7$ as suggested by Campbell [1985]. Of the algorithms that require an estimation of cloud cover, the lowest average RMSD over all sites was 26.2 W m$^{-2}$ for the Keding cloud correction combined with the Dilley clear-sky algorithm (Table 7). (The Crawford cloud correction stands out as being the best cloud correction when combined with the Ångström, Dilley or Prata clear-sky algorithms, but it does not require an estimate of cloud cover.) Using the cloud-estimation approach of Kasten and Czeplak [1980], the lowest average RMSD was 32.6 W m$^{-2}$ produced by a combination of the Dilley clear-sky algorithm and the Keding cloud correction. This is significantly larger than the RMSD produced using the approach of Campbell [1985]. The cloud-estimation approach of Kasten and Czeplak [1980] was therefore not considered further.

[28] Assumptions regarding cloud cover ($c$) and clearness index ($k$) were explored for the cloud-estimation approach of Campbell [1985] using the best clear-sky algorithms and cloud corrections from Table 7. Combinations not significantly different from the Dilley-Kimball combination were Ångström-Brutsaert, Ångström-Keding, Ångström-Maykut, and Dilley-Unsworth. Although the Prata algorithm was consistently one of the better clear-sky algorithms, these cloud corrections combined with the Prata clear-sky algorithm had significantly higher RMSD’s than the Dilley-Kimball combination. Values of $k_{clr}$ and $k_{cld}$ in Campbell’s cloud-estimation approach were evaluated for all combinations of these three clear-sky algorithms and five cloud corrections.

[29] Table 8 gives the RMSD of the Dilley-Kimball combined algorithm for all sites with various values of $k_{cld}$ and $k_{clr}$. The best combination for clear skies and complete cloud cover was $k_{cld} = 0.25$ and $k_{clr} = 0.8$. This yielded a significantly lower RMSD than the original clearness indices of 0.4 and 0.7 in Table 7. Additionally, these optimum limits in clearness index (0.25, 0.8) did not have a significantly higher RMSD for any of the sites than the best combination for that site. The limits of 0.25 and 0.8 correspond closely with the limits in observed $k$ for most sites. It is interesting to note that the cloud-estimation algorithm of Kasten and Czeplak [1980] yields similar limits in $k$ for zero and complete cloud cover (0.2, 0.8) as the optimum limits (0.25, 0.8) found for the Dilley-Kimball combination in Table 8. However, the simple linear interpolation between $k_{cld}$ and $k_{clr}$ of the approach of Campbell [1985] yielded better results than the nonlinear approach of Kasten and Czeplak [1980].
Table 9. Optimum Values of $k_{cd}$ and $k_{cl}$ for Zero and Complete Cloud Cover to Minimize the RMSD of Various Combinations of the Best Clear-Sky Algorithms and Cloud Corrections

<table>
<thead>
<tr>
<th>Clear-Sky Algorithm</th>
<th>Brutsaert</th>
<th>Kimball</th>
<th>Keding</th>
<th>Maykut</th>
<th>Unsworth</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ångström $k_{cd}$</td>
<td>0.25</td>
<td>0.40</td>
<td>0.25</td>
<td>0.15</td>
<td></td>
</tr>
<tr>
<td>Ångström $k_{cl}$</td>
<td>0.25</td>
<td>0.35</td>
<td>0.25</td>
<td>0.20</td>
<td></td>
</tr>
<tr>
<td>Dilley $k_{cd}$</td>
<td>0.25</td>
<td>0.40</td>
<td>0.30</td>
<td>0.15</td>
<td></td>
</tr>
<tr>
<td>Dilley $k_{cl}$</td>
<td>0.25</td>
<td>0.35</td>
<td>0.25</td>
<td>0.20</td>
<td></td>
</tr>
<tr>
<td>Prata $k_{cd}$</td>
<td>0.25</td>
<td>0.35</td>
<td>0.25</td>
<td>0.20</td>
<td></td>
</tr>
<tr>
<td>Prata $k_{cl}$</td>
<td>0.25</td>
<td>0.35</td>
<td>0.25</td>
<td>0.20</td>
<td></td>
</tr>
</tbody>
</table>

Table 10. Root Mean Square Deviations in Estimated Atmospheric Long-Wave Radiation (W m$^{-2}$) of All-Sky Algorithms for Each Site Using the Optimum Values for $k_{cd}$ and $k_{cl}$ From Table 9

<table>
<thead>
<tr>
<th>Site</th>
<th>Days</th>
<th>Ångström-Brutsaert</th>
<th>Ångström-Kimball</th>
<th>Ångström-Unsworth</th>
<th>Ångström-Crawford</th>
<th>Dilley-Brutsaert</th>
<th>Dilley-Kimball</th>
<th>Dilley-Unsworth</th>
<th>Dilley-Crawford</th>
<th>Prata-Crawford</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arizona</td>
<td>721</td>
<td>17.7</td>
<td>18.5</td>
<td>17.8</td>
<td>17.7</td>
<td>19.2</td>
<td>17.6</td>
<td>17.4</td>
<td>21.2</td>
<td>16.2</td>
</tr>
<tr>
<td>BC</td>
<td>713</td>
<td>*23.4</td>
<td>*23.3</td>
<td>*22.8</td>
<td>*22.9</td>
<td>*23.5</td>
<td>*23.5</td>
<td>*22.7</td>
<td>*23.1</td>
<td>*23.0</td>
</tr>
<tr>
<td>Colorado</td>
<td>714</td>
<td>*27.9</td>
<td>*28.0</td>
<td>*27.5</td>
<td>*27.3</td>
<td>*27.1</td>
<td>*26.0</td>
<td>*26.3</td>
<td>*26.1</td>
<td>*27.1</td>
</tr>
<tr>
<td>Ganzi</td>
<td>329</td>
<td>*26.6</td>
<td>*25.3</td>
<td>*25.3</td>
<td>*24.9</td>
<td>29.3</td>
<td>27.2</td>
<td>26.3</td>
<td>28.5</td>
<td>28.0</td>
</tr>
<tr>
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<td>708</td>
<td>26.9</td>
<td>26.2</td>
<td>25.7</td>
<td>25.8</td>
<td>25.6</td>
<td>24.1</td>
<td>23.9</td>
<td>24.6</td>
<td>25.1</td>
</tr>
<tr>
<td>Indiana</td>
<td>708</td>
<td>24.0</td>
<td>25.1</td>
<td>*21.8</td>
<td>22.2</td>
<td>24.9</td>
<td>23.8</td>
<td>*21.2</td>
<td>23.0</td>
<td>21.7</td>
</tr>
<tr>
<td>Iowa</td>
<td>546</td>
<td>*23.8</td>
<td>*23.5</td>
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<td>*22.2</td>
<td>*23.4</td>
<td>*22.2</td>
<td>*21.2</td>
<td>*23.0</td>
<td>*21.7</td>
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<tr>
<td>Mississippi</td>
<td>669</td>
<td>30.5</td>
<td>31.1</td>
<td>29.5</td>
<td>29.3</td>
<td>30.0</td>
<td>27.5</td>
<td>28.4</td>
<td>25.9</td>
<td>29.3</td>
</tr>
<tr>
<td>Nagqu</td>
<td>260</td>
<td>*28.9</td>
<td>*29.4</td>
<td>*30.3</td>
<td>*31.0</td>
<td>*30.1</td>
<td>*29.6</td>
<td>*31.0</td>
<td>*30.9</td>
<td>*31.4</td>
</tr>
<tr>
<td>North Carolina</td>
<td>688</td>
<td>29.7</td>
<td>32.0</td>
<td>29.5</td>
<td>30.0</td>
<td>*27.9</td>
<td>28.5</td>
<td>*28.0</td>
<td>*25.9</td>
<td>28.9</td>
</tr>
<tr>
<td>South Dakota</td>
<td>611</td>
<td>24.1</td>
<td>24.4</td>
<td>*23.8</td>
<td>*23.7</td>
<td>23.6</td>
<td>*22.6</td>
<td>*22.7</td>
<td>*22.6</td>
<td>*23.1</td>
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<tr>
<td>Washington</td>
<td>727</td>
<td>25.2</td>
<td>*23.8</td>
<td>*23.0</td>
<td>24.1</td>
<td>25.2</td>
<td>*24.0</td>
<td>21.8</td>
<td>25.6</td>
<td>24.9</td>
</tr>
<tr>
<td>Wisconsin</td>
<td>607</td>
<td>26.8</td>
<td>*26.4</td>
<td>*24.4</td>
<td>*25.0</td>
<td>27.1</td>
<td>25.6</td>
<td>*23.9</td>
<td>*25.1</td>
<td>*25.2</td>
</tr>
<tr>
<td>Yucheng</td>
<td>508</td>
<td>27.3</td>
<td>32.3</td>
<td>28.1</td>
<td>28.1</td>
<td>26.4</td>
<td>27.5</td>
<td>27.7</td>
<td>*23.4</td>
<td>27.1</td>
</tr>
</tbody>
</table>

All sites 7920

RMSD 25.9 26.4 25.1 25.3 26.0 25.0 24.5 24.9 25.1
RMSD 0.2 *10.5 12.5 *10.8 12.7 12.8 13.4 *10.7 14.0 11.7
RMSD 23.7 23.2 22.7 21.9 22.6 *21.1 22.0 *20.6 22.2
SD *60.0 55.2 57.4 54.9 61.9 55.1 *58.1 55.6 57.4
RMSDdaily 16.8 17.0 *15.4 15.6 17.4 *15.6 *14.9 *15.7 *15.1

* Asterisks indicate that the RMSD is not significantly different at the p = 0.05 level from the minimum RMSD for that site or that the SD of predicted values is not significantly different from the observed standard deviation of 59.0 W m$^{-2}$. 

[30] The best combination of $k_{cd}$ and $k_{cl}$ for the three best clear-sky algorithms and the five best cloud corrections that use cloud cover $c$ are presented in Table 9 based on an analysis similar to that in Table 8. Values for $k_{cd}$ and $k_{cl}$ selected in Table 9 for each combination of algorithms yielded an RMSD value for each site that was not significantly higher than for the optimum values of $k_{cd}$ and $k_{cl}$ for that site. The optimum limit for complete cloud cover, $k_{cdp}$, was largely independent of the clear-sky algorithm used and a value was selected for each cloud cover correction (Table 9). The optimum value for $k_{cl}$ varied with each combination of clear-sky algorithm and cloud cover correction.

4.4. Cloudy Sky Corrections

[31] The RMSD and MBD for combinations of the best clear-sky algorithms and cloud corrections using the values for $k_{cd}$ and $k_{cl}$ in Table 9 are given in Tables 10 and 11. (The RMSD’s using the Keding and Maykut cloud corrections were significantly higher for nearly all sites and were therefore omitted from the tables for brevity. Although the Prata clear-sky algorithm was not significantly higher for many of the sites when combined with the Brutsaert, Kimball, or Unsworth cloud corrections, the averages for all sites were significantly higher, and it was therefore omitted for these three cloud corrections.) The Keding and Brutsaert cloud corrections occasionally yielded emissivities above 1.0; this occurred most often when combined with the Prata clear-sky algorithm, for which 1% and 3% of all emissivity estimates, respectively, were above 1.0. While summaries in Tables 10 and 11 include emissivities greater than 1.0 for these two cloud corrections, a separate analysis limiting predicted emissivities to 1.0 had little to no influence on the overall RMSD and MBD (maximum change of 0.2 W m$^{-2}$ for the Dilley-Brutsaert combination).

[32] The algorithms that stand out as giving the best estimates for all sites are the Dilley clear-sky algorithm combined with the Kimball, Unsworth, or Crawford cloud correction and the Prata-Crawford combination. However, at the $p = 0.01$ level, the Unsworth and Crawford cloud corrections are not significantly higher when combined with any of the three best clear-sky algorithms. Of these combinations, the Ångström-Unsworth and Dilley-Unsworth yielded the lowest RMSD (Table 10). Apparently optimizing the clearness indices in the Unsworth cloud correction compensated for the negative MBD and relatively high RMSD of the Dilley clear-sky algorithm, giving this combination a relatively low RMSD. The lowest RMSD was produced by the Dilley-Kimball algorithm.

[33] The four all-sky algorithms by Aubinet underestimated $L_d$ for nearly all sites. MBD over all sites for the Aubinet algorithms was $-4.3$, $-15.7$, $-21.4$, and $-10.7$ W m$^{-2}$, respectively; RMSD was $29.5$, $30.1$, $36.0$ and $27.8$ W m$^{-2}$. The only sites that the RMSD of any of the Aubinet algorithms was not significantly higher than the best algorithm were Yucheng, Mississippi, and North Carolina.
which are the sites that the best algorithms tended to overestimate $L_d$. These are same three sites that the clear-sky algorithms overestimated $L_d$.

4.5. Influence of Length of Solar Window for Estimating Clearness Index

[34] The previous analyses were based on clearness index being estimated using a 24-hour window of solar radiation centered on the 30-minute or hourly observation. More accurate estimates of $L_d$ can likely be obtained, particularly for daytime values, by using a smaller window of observation. However, noontime values of $L_d$ based on a single solar radiation value corresponding to the $L_d$ observation yielded RMSD values higher than those using a 24-hour window of solar radiation. RMSD of noon values for $L_d$ averaged over all sites for the five best algorithms in Table 10 using a single solar observation ranged from 20.4 to 22.3 W m$^{-2}$ compared to 19.6 to 20.7 W m$^{-2}$ using a 24-hour window. RMSD values using a single solar observation were significantly higher for all but the Prata-Crawford combination. The reason for significantly higher RMSD values is likely due to the fact that clouds covering the solar disk at the time of observation do not necessarily represent average conditions over the entire sky hemisphere. RMSD averaged over all sites reached a minimum when a 4-hour window for solar radiation around the $L_d$ observation was used; RMSD values using the 4-hour window ranged from 19.3 to 20.6 W m$^{-2}$, which were not significantly lower than values using a 24-hour window of solar radiation. There was very little change in the RMSD for noon values when the solar radiation window was increased beyond eight hours. Clearly, stretching the solar observation window beyond sunrise and sunset would have no effect.

[35] The optimum window of solar radiation for estimating 06:00 and 18:00 $L_d$ values was the first six to eight hours following sunrise and prior to sunset, respectively. However, this did not produce significantly lower RMSD’s than the 24-hour window. Using a 24-hour window centered on the long-wave radiation observation worked best for the midnight observation.

[36] Significantly better estimates of $L_d$ were obtained when the solar index was computed based on the average solar radiation over the period, rather than computing a clearness index for each solar observation, then computing an average clearness index over the solar observation window. This was likely due to the fact that small errors in observed solar radiation at low sun angles were weighted more heavily when averaging clearness index values than when averaging solar radiation values.

4.6. Diurnal Variation and Daily Average Estimates

[37] Monteith [1973] points out that many of the long-wave algorithms are based on average climatological conditions and may not be appropriate for diurnal variation. Indeed, the RMSD of predicted daily $L_d$ (RMSD$_{daily}$ in Tables 5 and 10) computed from daily average observations is considerably lower than predictions for the 30-minute and hourly observations. RMSD$_{daily}$ values of 13.9 W m$^{-2}$ for clear skies and 14.9 W m$^{-2}$ for all-sky conditions are very near the instrument accuracy of 12 W m$^{-2}$ reported by Michel et al. [2008]; thus, direct comparison between the very best algorithms is problematic for daily average fluxes.

[38] The algorithms were able to capture only a portion of the diurnal variation around the average $L_d$ flux for a given day. The root mean square variation of observed $L_d$ around the average flux for a day was 22.7 W m$^{-2}$. The fraction of the observed diurnal variation around the average daily flux explained by the algorithms was computed from

$$DE = 1 - \frac{\sum_{i=1}^{N} \sum_{j=1}^{M} [(P_{ij} - \bar{P}_i) - (O_{ij} - \bar{O}_j)]^2}{\sum_{i=1}^{N} \sum_{j=1}^{M} [O_{ij} - \bar{O}_j]^2}$$

where $DE$ is defined here as the diurnal efficiency and can range from negative infinity to 1.0. $P_i$ and $\bar{O}_j$ are the average predicted and observed fluxes for day $i$, and subscript $j$ refers to each 30-minute or hourly observation. $DE$ ranged from 0.21 to 0.28 for the best algorithms, indicating that the 30-minute or hourly estimates capture the diurnal variation slightly better than the average estimate for the day. Thus the algorithms have some capability for predicting diurnal variation, but it is marginal.
for the Mississippi site. Of that for hour 16:00 for most sites, the MBD ranged from 25.0 to 27.7 W m \(^2\) for the midnight estimates. Root Mean Square Deviations in Estimated Atmospheric Long-Wave Radiation (W m \(^2\)) were 0.05 level from the minimum RMSD for that site or that the SD of predicted values is not significantly different from the observed standard deviation of 63.4 W m \(^2\).

4.7. Daytime Versus Nighttime Estimates

For nearly all sites, daytime estimates of \(L_d\) were better than nighttime estimates. As indicated earlier, the RMSD of noon estimates ranged from 19.3 to 20.6 W m \(^2\) for the five best algorithms in Table 10 using a 4-hour solar window; midnight estimates of \(L_d\) ranged from 25.0 to 25.6 W m \(^2\). Greater daytime accuracy can be attributed to more uncertainty in cloud cover during nighttime compared to daytime and fewer surface inversions during daytime.

Notable exceptions to more accurate daytime estimates were observed around hour 16:00 at the Yucheng, North Carolina and Mississippi sites. Typical RMSD’s for 16:00 at these sites for the five best algorithms were 29 to 33 W m \(^2\) compared to 24 to 28 W m \(^2\) for the midnight values. Also, while MBD of midnight estimates were within 5 W m \(^2\) of that for hour 16:00 for most sites, the MBD increased by around 12 W m \(^2\) for the Yucheng and North Carolina sites and 24 W m \(^2\) for the Mississippi site. Interestingly, these are the same sites that were consistently overpredicted by the clear-sky algorithms (Table 6). MBD of all nighttime estimates for these sites using the five best algorithms are similar to the midnight hour, while the MBD for daytime estimates gradually increase from nearly no bias at around 6:00 to a maximum bias at around 15:00 to 16:00, then back to nearly no bias at around 20:00 to 21:00. Figure 1 displays the error of the Dilley-Unsworth algorithm, which is typical of the five best algorithms, with varying clearness index at hours 0:00 and 16:00. Inspection of this plot indicates that the daytime bias is more severe on relatively clear days (clearness index greater than 0.6) than on cloudy days. At full cloud cover (clearness index less than 0.2), estimates show very little error compared to measured values. However, the error is clustered around +40 W m \(^2\) when the clearness index exceeds 0.6. As is typical of all sites, there is more scatter in the error around a clearness index of 0.4; this is likely due to the fact that there is more uncertainty in cloud conditions around 50% cover than when solar radiation measurements indicate either obviously clear or cloudy skies. The bias in measured \(L_d\) at the Mississippi, North Carolina and Yucheng sites above a clearness index of 0.6 may suggest a bias in either long-wave radiation measurements or temperature measurements when there is strong solar radiation loading. Therefore the apparent bias in the long-wave radiation estimates for these three sites may be a problem with the instruments rather than a problem with the algorithms.

4.8. Test of Algorithms on Independent Sites

Table 12 summarizes the results for best algorithms tested on a set of sites independent of the optimized clearness indices. The Dilley clear-sky algorithm combined with either the Kimball, Unsworth, or Crawford cloud correction again yielded significantly lower RMSD than other algorithms for both half-hourly and daily observations. Interestingly, the Dilley-Crawford algorithm, which required no prior optimization of the clearness indices, had the lowest RMSD for nearly every site and one of the best MBD’s. This algorithm along with the Dilley-Kimball

Table 12. Root Mean Square Deviations in Estimated Atmospheric Long-Wave Radiation (W m \(^2\)) of the Best All-sky Algorithms Applied to a Set of Independent Sites

<table>
<thead>
<tr>
<th>Site</th>
<th>Days</th>
<th>Ångström-Brutsaert</th>
<th>Ångström-Unsworth</th>
<th>Ångström-Crawford</th>
<th>Dilley-Brutsaert</th>
<th>Dilley-Kimball</th>
<th>Dilley-Unsworth</th>
<th>Dilley-Crawford</th>
<th>Prata-Crawford</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alaska</td>
<td>348</td>
<td>*38.9</td>
<td>*38.2</td>
<td>*37.1</td>
<td>*36.4</td>
<td>*36.4</td>
<td>*35.7</td>
<td>*34.5</td>
<td>*34.7</td>
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\(^a\) Asterisks indicate that the RMSD is not significantly different from the observed standard deviation of 63.4 W m \(^2\).

\(^b\) Values for Alaska site include spring and summer data only.
algorithm also had significantly lower unsystematic errors. However, the Dilley-Crawford algorithm yielded estimates with significantly lower SD than the observed values.

[42] RMSD tended to be highest for the Alaska site and MBD was typically around $-10 \, \text{W m}^{-2}$. Estimates for the Alaska site were limited to spring and summer observations because the clearness index becomes undefined for a portion of the fall and winter months. Using a clearness index of 0.5 for the fall-winter months yielded an RMSD of 36.5 W m$^{-2}$ for the Dilley-Unsworth algorithm compared to 34.5 W m$^{-2}$ for the spring-summer months using the computed clearness index (Table 12) and 40.5 W m$^{-2}$ for the spring-summer months using a clearness index of 0.5.

[43] Estimates for the Montana site had somewhat higher RMSD compared to most other sites (Table 12) and tended to overpredict $L_{\text{clr}}$. MBD of the best algorithms ranged from $+8$ to $+16 \, \text{W m}^{-2}$ for the Montana site and the diurnal shift in bias exhibited the same trend as the Mississippi, North Carolina and Yucheng sites.

5. Discussion and Conclusions

[44] The Dilley, Prata and Ångström algorithms were the best clear-sky algorithms. The Dilley and Prata algorithms were the only ones based on precipitable water as a predictive factor, though the estimate of precipitable water is here based on a ratio of $e_0$ and $T_0$ [Prata, 1996], while the Ångström algorithm for clear-sky emissivity was based solely on vapor pressure. The Idso-l and Swinbank models, which are the only two models based solely on temperature, did the poorest. The study performed by Hatfield et al. [1983] also found that the temperature-based algorithms compared poorly to other algorithms. Hatfield et al. [1983], who did not evaluate the Dilley, Prata or Ångström algorithms, found a form of the Brutsaert equation performed best in their analysis of 15 locations; the Brutsaert algorithm ranked fourth in the present analysis based on RMSD.

[45] Efforts to include an elevation correction in the clear-sky algorithms proved fruitless. Extrapolating coefficients from two elevations provided by Iziomon et al. [2003] did not work outside the range observed in their study. The Iziomon algorithm performed reasonably at lower elevations, but was one of the worst algorithms for elevations above 1489 m, the elevation of their highest site. The elevation correction developed by Marks and Dozier [1979] for the Brutsaert algorithm reduced estimated $L_{\text{clr}}$ by an average of 40.7 W m$^{-2}$ for the highest elevation, however the Brutsaert algorithm already underpredicted $L_{\text{clr}}$ for nearly all sites. Marks and Dozier [1979] also noted a systematic underprediction of $L_{\text{clr}}$ using the elevation-corrected Brutsaert algorithm. The elevation correction suggested by Deacon [1970] appeared to overemphasize the effect of elevation. Correlations between MBD and elevation provided no evidence of an elevation effect. One might conclude that the error in the algorithms and uncertainty in the measurements may overshadow the influence of elevation on $L_{\text{clr}}$.

[46] The cloud-correction algorithms of Kimball, Unssworth and Crawford described the data best and are recommended for most sites. These performed best when combined with the Dilley clear-sky algorithm. Limits in the clearness index of 0.20 to 0.25 for complete cloud cover and 0.8 for clear skies are suggested herein for the Unsworth and Kimball cloud corrections; these cover a broader range in clearness index than used previously by others [Campbell, 1985; Flrechinger, 2000; Xiao et al., 2006] but correspond more closely with observed limits in clearness indices. The optimum clear-sky clearness index using the Dilley clear-sky algorithm tended to be higher than the other clear-sky algorithms, likely compensating for the fact the Dilley algorithm tended to underestimate $L_{\text{clr}}$.

[47] Uncertainty in cloud cover was a large contributor to the observed errors between estimated and observed $L_d$. More scatter in the error was observed around a clearness index of 0.4 to 0.5 and less when solar radiation measurements clearly indicated complete cloud cover or completely clear skies. Also, more scatter was observed in nighttime estimates because cloud corrections are based on daytime solar measurements. The optimum window of solar radiation measurements used to estimate cloud cover varied with the time of day. A 4-hour window proved best for noon estimates; the first six to eight after sunrise or before sunset gave the lowest RMSD for estimates around dawn and dusk; and a 24-hour window centered around the observed $L_d$ gave the best result for midnight observations. However, the optimum window for the noon estimates and estimates near dawn and dusk were not significantly better than using a 24-hour window.

[48] Although the algorithms do not specifically address cloud type, haze and turbidity of the atmosphere, these contributions to incident long-wave radiation are indirectly compensated for because the more they influence measured solar radiation, the more they will contribute to incident long-wave radiation. One might conjecture whether changes in atmospheric CO$_2$ have influenced atmospheric long-wave radiation, however Kessler and Jaeger [2003] found no marked change in a 27-year record in Germany. One clear limitation with the presented approaches is application during winter at extreme latitudes where cloud conditions must be based on very short or nonexistent daylight, as with the Alaska site.

[49] RMSD of daily estimates were as low as 14.9 W m$^{-2}$ compared to 24.5 W m$^{-2}$ for 30-minute and hourly estimates of $L_d$. Clearly the algorithms had difficulty capturing the diurnal variation (typically 22.7 W m$^{-2}$) around the daily average flux. A portion of the higher RMSD for 30-minute estimates can be attributed to more uncertainty in cloud conditions for 30-minute predictions, but the RMSD of 19.3 W m$^{-2}$ for noontime predictions when estimated cloud conditions would presumably have more certainty, is still appreciably higher than RMSD$_{\text{daily}}$ values in Table 10. Another likely cause for difficulty in predicting diurnal variation in $L_d$ is rapid air temperature changes with height near the surface due to diurnal heating and cooling.

[50] Observed RMSD for half-hourly or hourly estimates from the best algorithms over all sites (22.9 W m$^{-2}$) for clear-sky conditions and 24.5 W m$^{-2}$ for all-sky conditions, or about 8% of observed $L_d$ are comparable to other studies where algorithms were calibrated for specific sites. Lhomme et al. [2007] developed a relation for two sites in Bolivia that yielded an RMSD of 27 to 28 W m$^{-2}$; Marks and Dozier [1979] presented a relationship for clear-sky conditions in the southern Sierra Nevada Mountains with an RMSD of 26 to 31 W m$^{-2}$ over the two years of that
analysis. Alados-Alboledas et al. [1995] reported 6% accuracy for a modification of the Kimball algorithm for a site in Spain. Iziomon et al. [2003] also reported 6% accuracy for an algorithm calibrated for two sites in Germany. However, Niemelä et al. [2001] reported somewhat better results of 11.8 W m\(^{-2}\) RMSD for midnight estimates of \(L_d\) using a derived expression at a site in Finland. The lower RMSD reported by Niemelä et al. [2001] may be because they had the benefit of visual cloud observations and were not attempting to capture diurnal variation. Thus the algorithms recommended herein can be applied to a wide range of climates, elevations and latitudes with accuracy similar to that reported for algorithms calibrated for specific sites.

Appendix A

[51] Clear-sky solar radiation at the earth’s surface (\(S_{cb}\)) was calculated using a model presented by Crawford and Duchon [1999] based on the results of Paltridge and Platt [1976] and Meyers and Dale [1983] where instantaneous \(S_{cb}\) is approximated by

\[
S_{cb} = S_0 (\sin \alpha) \tau_R \tau_{pg} \tau_w \tau_a
\]  

(A1)

for positive solar altitude angles, \(\alpha\), which is the angle of the sun above the horizon. Here, \(S_0\) is the solar constant (taken as 1360 W m\(^{-2}\)) and \(\tau_R\), \(\tau_{pg}\), \(\tau_w\) and \(\tau_a\) are transmission coefficients for Rayleigh scattering, absorption by permanent gases, absorption by water vapor, and absorption and scattering by aerosols. The empirical expression for the product of the first two transmission coefficients is

\[
\tau_R \tau_{pg} = 1.021 - 0.084|m(0.00949P + 0.051)|^{0.5}
\]  

(A2)

where \(P\) is the pressure (kPa) and \(m\) is the optical air mass at 101.3 kPa given by \(m = 35\sin \alpha((1224\sin^2 \alpha + 1)^{-0.5} - 1\). The transmission coefficient for absorption by water vapor is

\[
\tau_w = 1 - 0.077wm^{0.3}
\]  

(A3)

where \(w\) is precipitable water defined in Table 1. The transmission coefficient for aerosols is

\[
\tau_a = 0.935^n.
\]  

(A4)

The solar altitude angle is computed from

\[
\sin \alpha = \sin \lambda \sin \delta + \cos \lambda \cos \delta \cos (\pi(t - t_{noon})/12)
\]  

(A5)

where \(\lambda\) is the latitude of the site, \(\delta\) the solar declination, and \(t_{noon}\) is the time of solar noon. Solar declination is computed from

\[
\delta = 0.4012 \sin \left(\frac{2\pi}{365}(J - 80)\right).
\]  

(A6)

where \(J\) is the day of the year. Values for \(S_{cb}\) for each 30-minute or hourly observation of \(L_d\) were obtained from averaging equation (A1) over a 24-hour period surrounding the observation.

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References


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